Calibration for non-positive definite covariance matrix

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Abstract

Covariance matrices that fail to be positive definite arise often in covariance estimation. Approaches addressing this issue exist, but are not well supported theoretically. In this paper, we propose a unified statistical and numerical matrix calibration method, finding the optimal positive definite surrogate in the sense of Frobenius norm. The proposed method is well supported theoretically and the proposed algorithm can be directly applied to any estimated covariance matrix. Numerical simulation results show that the calibrated matrix is typically closer to the true covariance, while making only limited changes to the original covariance structure. The proposed method is also applied to a real data analysis for illustration.

Keywords

Covariance matrix calibration, Nearness problem, Non-positive definiteness, Spectral decomposition

References

[1] Diggle, P. J. (1988). An approach to the analysis of repeated measures. *Biometrics*, **44**, 959-971.

[2] Diggle, P.J. and Verbyla, A. P. (1998). Nonparametric estimation of covariance structure in longitudinal data. *Biometrics*, **54**, 401-415.

[3] Higham, N.J. (1988). Computing a nearest symmetric positive semidefinite matrix. *Linear Algebra and Appl.*, **103**, 103-118.

[4] Huang C., Farewell D. and Pan J. (2017). A calibration method for non-positive definite covariance matrix in multivariate data analysis. *Journal of Multivariate Analysis*, **157**, Pages 45-52.

[5] Zarowski., Christopher J. (2004). An Introduction to Numerical Analysis for Electrical and Computer Engineers. Wiley.