A Wicksell-Kibble Type Distribution on a Hyper-Cylinder with an Application to Wind Direction and Speed Data

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Abstract

Observation values of wind speed (linear) at 6:00 a.m. and 12:00 noon may be modeled by bivariate gamma, lognormal, and inverse Gaussian distributions. One can predict wind speed values at noon from data at 6:00 using a regression and furthermore may expect a better statistical modeling if wind direction (circular) data at 6:00 are available. In this talk we use the Wicksell-Kibble distribution as a bivariate gamma distribution to construct a hyper-cylindrical distribution with two linear and one circular variables.

The Wicksell-Kibble bivariate gamma distribution considered here has four-parameters with the role of shape (one), scale (two), and correlation coefficient (one). The proposed hyper-cylindrical distribution involves one more parameter as a circular location. The parameter of correlation coefficient for the Wicksell-Kibble distribution controls not only dependence of the three variables but also circular concentration for the hyper-cylindrical distribution. Several properties such as marginal and conditional distributions and their moments are studied, and the regression curve and surface are obtained. Random number generation for the proposed distribution is possible as the joint density is expressed by a multiplication of conditional and marginal densities, and random numbers for each of conditional and marginal distributions can be generated. In particular, the conditional density of one linear variable given other linear and circular variables has a mixture expression of a Poisson probability and a gamma density.

An illustrative example is given for wind direction and speed at 6:00 a.m. and wind speed at 12:00 noon data observed in Tokyo. A comparison between the proposed and existing models is made. It is clear that the model has potential applications to a combination of any one circular and two linear measurements.

Keywords

Directional statistics, Random number generation, Regression surface