ANOVA SSs and Proportional Subclass Numbers

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Abstract

Soon after Fisher introduced analysis of variance for effects of two factors, it was clear that "the addition law" didn't work in unbalanced models unless the cell sample sizes had the "proportional subclass numbers" property (psn), that $n_{ij} = n_{i\cdot}n_{\cdot j}/n_{\cdot\cdot}$. If not, then SS_{AB}, computed as Fisher described, was not a true SS in the usual sense, and it could take negative values. This led to the still-continuing ambivalence about the appropriate SSs for testing factor main effects in unbalanced models without psn. Consistently, though, textbooks have taught that there is no problem in models having psn: psn is the same as balanced.

In this talk I'll note that this is not true. In unbalanced models that don't have psn, the classical ANOVA SSs test hypotheses that are unrelated to the ANOVA definition of main effects. SS_{AB} tests the right hypothesis iff the model has psn. SS_A tests the right hypothesis iff $n_{ij} = n_{i\cdot}/b$, an additional requirement beyond psn.

The process of examining these properties reveals relations among Types I, II, III, and marginal-means (MM) SSs. For example, the Type II noncentrality parameter for A main effects can be 0 even though there are differences (arbitrarily great) among the A marginal means. Type III SSs, on the other hand, always test at least the estimable part of the corresponding effect contrasts, and MM SSs test exactly the estimable part. These are illustrated by examples.

Keywords

Unbalanced two-factor models, Orthogonal sums of squares, Type I-III SSs.